Lesson 2-1: Conditional Statements

Where we are headed

Thus far we've been exploring the building blocks of geometry. Our exploration has primarily been using inductive reasoning. Do you recall what it means to reason inductively?

Inductive reasoning is used when you recognize a pattern, and form a conjecture (hypothesis) based on your observations. Inductive reasoning is one of the most used ways of reasoning in mathematics. But inductive reasoning has a very important limitation; do you recall what it is? You can't prove your conjecture with inductive reasoning; you can only disprove it by finding a counter-example.

In order to prove a conjecture true, you will need to use a different form of reasoning: deductive reasoning. Deductive reasoning is the core of geometric proof. In this chapter you will learn how to reason deductively and how a geometric proof is done.

Before we get going, take a minute and read the "*Where You're Going"* section and the "*Chapter 2 Lessons*" list on page 67. This will help you put what we're talking about in some context.

The core of deductive reasoning – the conditional statement

The core of deductive reasoning is in something you use every day: the simple *if-then* statement. The *if-then* statement, also known as the *conditional* statement, has two parts in the following form:

> **if *hypothesis* then *conclusion***

Here is an example:

> if it is raining then water is falling from the sky

Can you pick out the hypothesis and conclusion? The hypothesis is ***it is raining*** and the conclusion is ***water is falling from the sky***. Try it out on *Check Understanding 1* on page 68.

> The hypothesis is $y - 3 = 5$.
> The conclusion is $y = 8$.

Writing a conditional

You can go the other way and build a conditional statement from a statement. Take a look at problem #12 on page 71:

> All obtuse angles have measure greater than 90.

Break the statement into two parts. The first part could be "all obtuse angles" and the second "measure greater than 90." Determine the subject of the first part and turn it into general reference. Then use the first as the hypothesis and the second as the conclusion:

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if an angle is an obtuse angle then it has a measure greater than 90.

Proving a conditional statement false

The easiest way to prove a conditional false is to find a counter-example. Take a look at problem #18 on page 72. The conditional is

if you play a sport with a ball and a bat then you are playing baseball

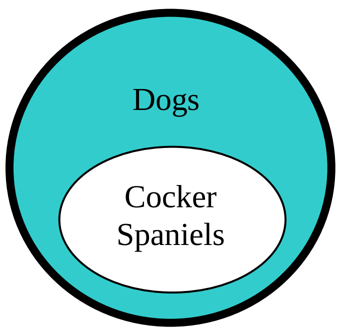
With just a little thought, you can think of a sport that uses a ball and bat that isn't baseball. Two examples are softball and cricket.

Truth values

A conditional is a pretty simple thing; it has two and only two options for its value, true or false. This is referred to as the conditional's truth value. If you are asked to determine a conditional's truth value, all you are being asked is "is the conditional true or false?" To show it false, simply find a counter-example.

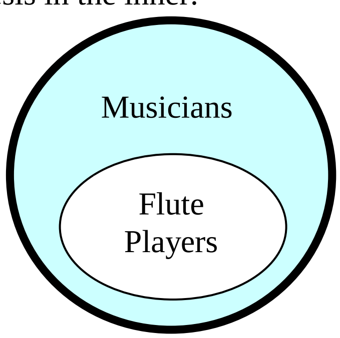
A way to visualize conditionals

A very good way to visualize a conditional is with a Venn diagram. Can you guess what conditional the following Venn diagram represents?



If something is a cocker spaniel then it is a dog. The inner circle becomes the hypothesis, the outer becomes the conclusion.

Likewise, you can draw a Venn diagram from a conditional. Consider problem #20 on page 72. "If you play the flute, then you are a musician." Identify the hypothesis (you play the flute) and make a generalized version of it ("flute players"). Do the same with the conclusion ("you are a musician" → "musicians"). The conclusion will be in the outer circle and the hypothesis in the inner:



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Playing around with a conditional

Once we define and understand something, it is interesting to play with it – tweak it a bit and see what we come up with.

Converse

Considering conditionals, an interesting question is what would happen if you took a conditional and swapped the hypothesis and conclusion? Before we go any further, let's name this new thing: a **converse**.

Now, assuming the original conditional is true, will the converse always be true? Hmm, proving that will be rather hard. Maybe it would be easier to prove it wrong? Let's look for a counter-example. Pick a conditional, any conditional; let's use our earlier example:

> **if it is raining then water is falling from the sky**

I think we can all accept this conditional as true. What about the converse?

> **if water is falling from the sky then it is raining**

Is the converse true? Asked another way, what is the truth value of this converse? Look for a counter-example: can you think of a situation in which water is falling from the sky and it isn't raining? Sure! If I'm watering the lawn and spraying water into the air, then water is falling from the sky. So this converse is false and we have found a counter-example proving our hypothesis wrong. Thus (or we will often say therefore in math), therefore, even if a conditional is always true, the converse is not necessarily true.

Example – p. 72 #28

Write the converse of the statement and determine the truth value of the original conditional and of the converse.

> If a point is in the first quadrant, then its coordinates are positive.

First, the converse:

> If the coordinates of a point are positive, then it is in the first quadrant.

Now the truth values:

> Conditional: true
> Converse: true

Writing conditionals symbolically

In math we love to find very concise ways of writing things. The same holds true for logic; we have a very simple way of writing a conditional using symbols. We use the letter p to stand for the hypothesis, and the letter q to stand for the conclusion. We can then write $p \rightarrow q$ to mean if p then q. Can you guess what the converse would be? It would be $q \rightarrow p$.

Often what you will see is something like:

> Let p: The point is in the first quadrant.
> Let q: The point's coordinates are positive.
> $p \rightarrow q$ (the conditional)
> $q \rightarrow p$ (the converse)

Postulates as conditionals

You can also take a postulate as a conditional. Consider problem #54 on page 73:

> Two intersecting lines meet in exactly one point. (Postulate 1-2)

Now as a conditional:

> If two lines intersect, then they meet in exactly one point.

Assign homework

> p. 71 #1-35 odd, 41-47 odd, 51, 55-61
> p. 66 #1-30